Celebration of Chris Pope's Contributions to Physics

Cook's Branch,
September 27-30, 2023

I am honored and delighted to be able to participate in the celebration of Chris's numerous contributions theoretical physics, primarily gravitational physics and supersymmetry, including his fundamental contributions to supergravity theories, Kaluza Klein compactification, and broad aspects of black hole physics.

I cherish him as a friend, a colleague and a close collaborator. These fruitful collaborations were initiated with the arrival of Hong Lü at UPenn in 1998 and an especially memorable time spent at Henri Poincaré Institute in the Fall of1999.

Our scientific ties continued during my many visits at Texas A&M, his visits at Penn, and summers in Trieste, CERN, Benasque…and later at Cook's Branch, Cambridge…where we had the opportunity to not only collaborate but also to explore wining and dining scenes in Philadelphia, Paris, Geneva, Trieste…and Chris's home in College Station.

I was impressed and intimidated by his intensity and impossibility to compete with his calculational skills…No-one in the world can calculate a spin-connections with the speed and precision as Chris does…So, I gave up on competing with him on that a long time ago…

He is a generous and patient collaborator, and a loyal and supportive friend.

Our association over past 25 years resulted in 81 papers (with over 6200 citations).

This collaboration spans many aspects of gravitational physics, with primary applications to supergravity and string theory as a common thread: special holonomy spaces, non-linear Kaluza-Klein reduction in supergravity theories, and extensive work on black holes there.

In addition to numerous works together with Hong Lü, it also led to collaborations with other colleagues: Kelly Stelle, Gary Gibbons, James Liu, Mike Duff, Don Page, Bernard Whiting, Sera Cremonini… & numerous Chris's students and post-docs and some of mine.

Highlights:

- Non-linear consistent Kaluza-Klein compactifications on spheres and other spaces:

M. Cvetič, H. Lü and C.N. Pope, ``Gauged six-dimensional supergravity from massive type IIA," Phys. Rev. Lett. 83, 5226-5229 (1999) arXiv:hep-th/9906221.

M. Cvetič, M.J. Duff, P. Hoxha, J.T. Liu, H. Lü, J.X. Lu, R.Martinez-Acosta, C.N. Pope, H. Sati and T.A.Tran, ``Embedding AdS black holes in ten-dimensions and eleven-dimensions," Nucl. Phys. B 558, 96-126 (1999) arXiv:hep-th/9903214 [hep-th]

...

- Explicit metrics of special holonomy spaces, including G_2 and Spin(7):

M. Cvetič, G.W. Gibbons, H. Lü and C.N. Pope, ``New complete noncompact spin(7) manifolds," Nucl. Phys. B 620, 29-54 (2002) arXiv:hep-th/0103155 [hep-th]

….

- New metrics of Einstein Sasaki spaces:

M. Cvetič, H. Lü, D.N. Page and C.N. Pope, ``New Einstein-Sasaki spaces in five and higher dimensions," Phys. Rev. Lett. 95, 071101 (2005) arXiv:hep-th/0504225 [hep-th]

…

- New Black holes and their physics

Z.W. Chong, M. Cvetič, H. Lü and C. N. Pope, ``General non-extremal rotating black holes in minimal five-dimensional gauged supergravity,'' Phys. Rev. Lett. 95, 161301 (2005) arXiv:hep-th/0506029 [hep-th].
…

M. Cvetič, G. W. Gibbons and C. N.Pope, ``Universal Area Product Formulae for Rotating and Charged Black Holes in Four and Higher Dimensions,'' Phys. Rev. Lett. 106, 121301 (2011) arXiv:1011.0008 [hep-th]
…

and it continues….

M. Cvetič, G.W. Gibbons, C.N. Pope and B.F. Whiting, ``Positive Energy Functional for Massless Scalars in Rotating Black Hole Backgrounds of Maximal Ungauged Supergravity,'' Phys. Rev. Lett. 124, no.23, 231102 (2020) arXiv:1912.08988 [gr-qc]
…

S. Cremonini, M. Cvetič, C.N. Pope and A. Saha, ``Mass and force relations for Einstein-Maxwell-dilaton black holes,'' Phys. Rev. D 107, no.12, 126023 (2023) arXiv:2304.04791 [hep-th]
…

Higher-Form Symmetries in (Non-)Compact M-/F-Theory

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Motivation

M/string theory on singular special holonomy spaces X:

- Non-compact spaces X →
  Geometric engineering of supersymmetric quantum field theories (SQFTs):

- Build dictionary:{operators, symmetries} ⟺ {geometry,topology}
- Focus: higher-form global symmetries ⟺ topology
  (associated with ``flavor'' branes)

- Compact spaces X →
  Quantum field theory (QFT) w/ gravity →
  Higher-form symmetries gauged or broken
  Physical consistency conditions → swampland program

Higher-form symmetries in (S)QFT - active field of research

[Gaiotto, Kapustin, Seiberg, Willet, 2014],…

Higher-form symmetries & geometric engineering

[Del Zotto, Heckman, Park, Rudelius, 2015],…
[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020],…
[M.C., Dierigl, Lin, Zhang, 2020],..
[Apruzzi,Bhardwaj, Oh, Schäfer-Nameki, 2021],…
[M.C., Dierigl, Lin, Zhang, 2021],…
[M.C., Heckman, Hübner, Torres, 2203.10102],
[Del Zotto, Garcia Etxebarria, Schäfer-Nameki, 2022],…
[Hübner, Morrison, Schäfer-Nameki,Wang, 2022],…
[Heckman, Hübner, Torres, Zhang, 2023],…
[M.C.,Heckman, Hübner, Torres, Zhang, 2023],..

Higher-form symmetries & and compact geometry

[M.C., Dierigl, Lin, Zhang, 2020, 2021,2022],…
[M.C., Heckman, Hübner, Torres, 2307.1023],…

Goals

- Identify geometric origin of higher-form symmetries for M-/string theory non-compact special holonomy spaces X

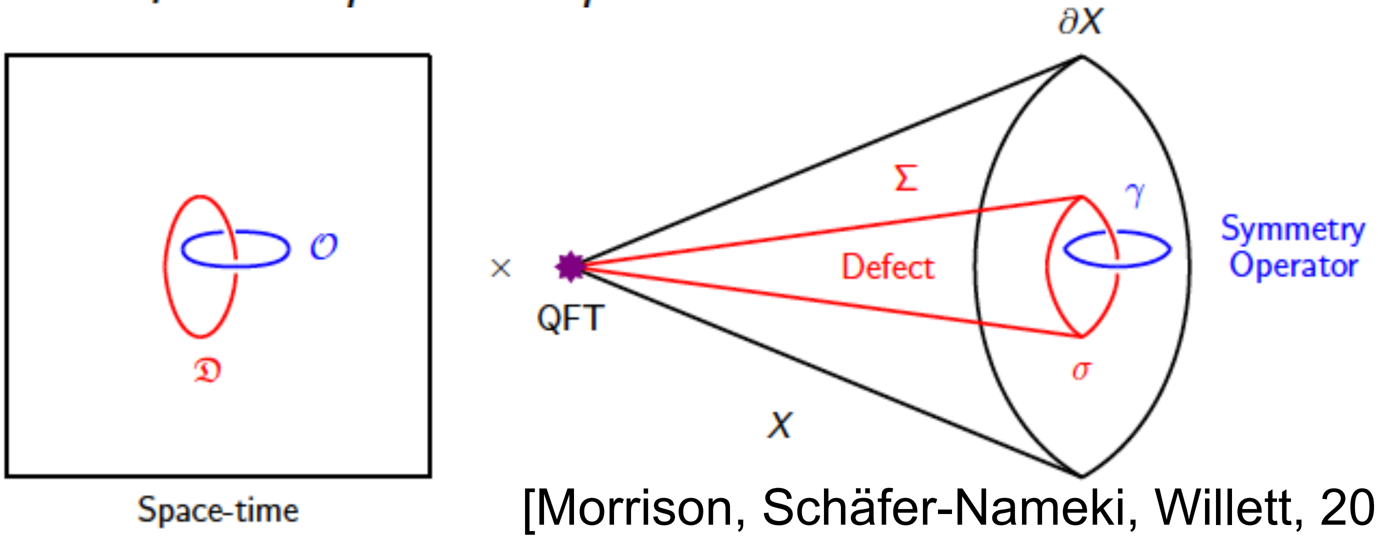
  Punchline: 0-form,1-form and 2-group symmetries via cutting and gluing of singular boundary geometries

- Examples: M-theory on non-compact Calabi-Yau n-folds (n=2,3..) [orbifolds & elliptically fibered CYs]

  New results for compact geometries [compact elliptic examples – dual to F-theory compactification]

I. Defect group for M-theory on non-compact X

- Defect Group for extended p-dim operators associated with M2 and M5 branes: $\mathcal{D}_p = \mathcal{D}_p^{\mathrm{M2}} \oplus \mathcal{D}_p^{\mathrm{M5}}$
- Schematically:



[Morrison, Schäfer-Nameki, Willett, 2020], [Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020]

w/ M2, M5 in X wrapping relative cycles

$$\mathcal{D}_p^{\mathrm{M2}} = \frac{H_{3-p}(X,\partial X)}{H_{3-p}(X)} \cong H_{3-p-1}(\partial X)|_{\mathrm{triv}}$$ [p-dim el. operators in SCFT]

$$\mathcal{D}_p^{\mathrm{M5}} = \frac{H_{6-p}(X,\partial X)}{H_{6-p}(X)} \cong H_{6-p-1}(\partial X)|_{\mathrm{triv}}$$ [p-dim mag. operators in SCFT]

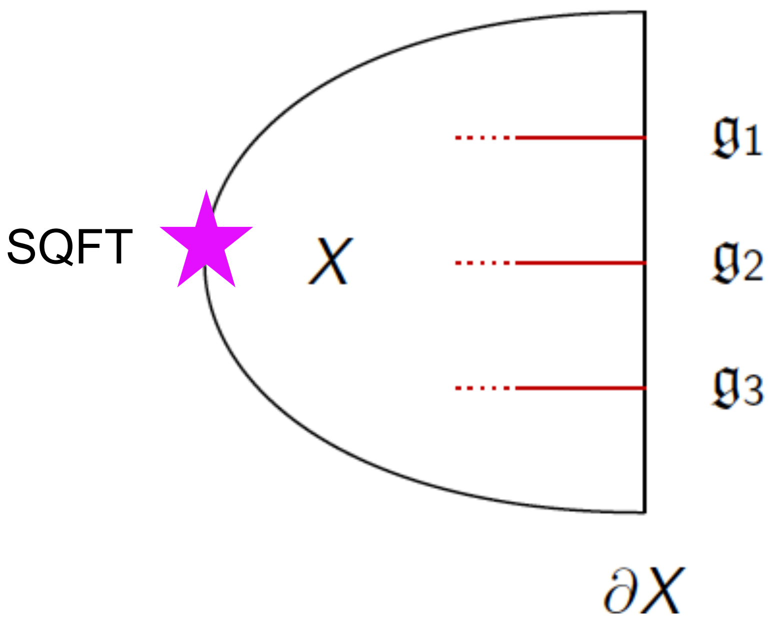
[focus on torsional cycles]

- Focus on defect ops.; symmetry ops.c.f. [Heckman, Hübner, Torres, Zhang2022,..; M.C., Heckman, Hübner, Torres, Zhang 2023]

II. Geometrizing Topology of Flavor Group

Non-compact ADE loci ≡ flavor branes → flavor symmetries

Naïve flavor symmetry $\widetilde{G}_F$ (simply connected w/ Lie Algebra $\mathfrak{g}_i$)



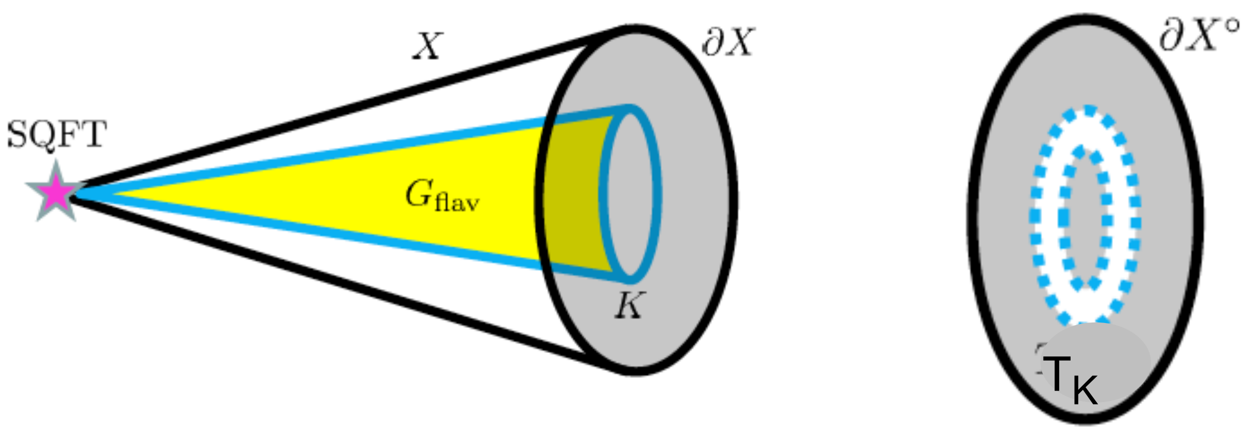
[From now on only ADE's in ∂X]

$$\widetilde{G}_F = \widetilde{G}_1 \times \widetilde{G}_2 \times \widetilde{G}_3 \times \dots$$

(Flavor Wilson) lines → fix topology of favor symmetry G_F
from singular boundary topology

III. Boundary geometry of flavor branes:

- Singular non-compact space X w/
  $K = \cup_i K_i$ - ADE loci (of flavor branes) in the boundary ∂X
- Define a smooth boundary $\partial X^\circ = \partial X \setminus K$
  & a tubular region T_K (excise K)
- Locally $T_K \cap \partial X^\circ \cong \cup_i K_i \times S^3/\Gamma_i$

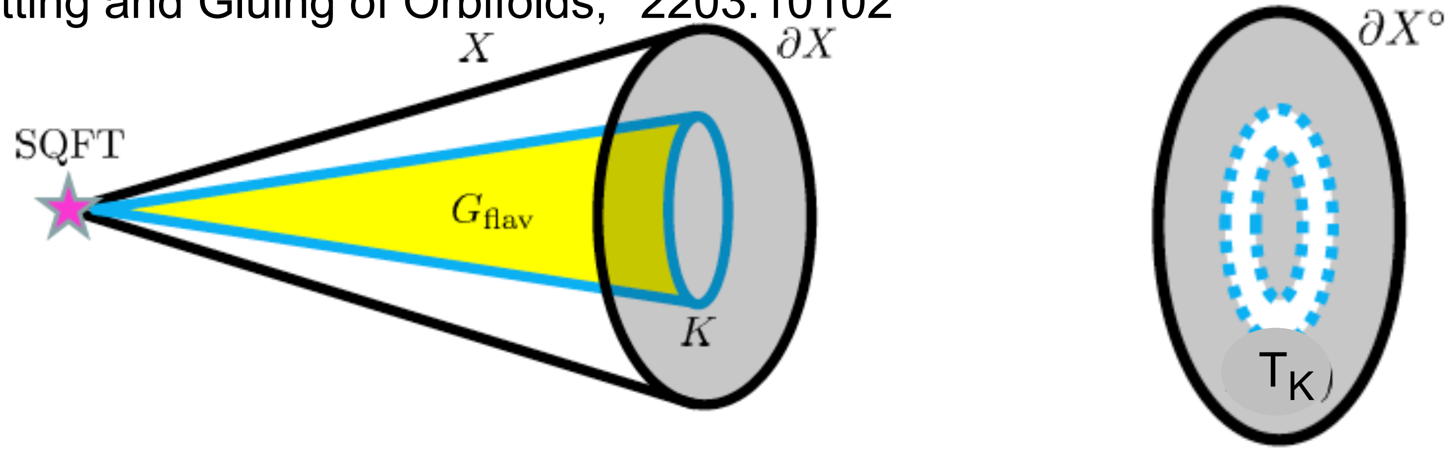


- Naïve flavor center symmetry:

$$Z_{\widetilde{G}_F} = \text{Tor}\, H_1(T_K \cap \partial X^\circ) \cong Z_{\widetilde{G}_1} \oplus Z_{\widetilde{G}_2} \oplus Z_{\widetilde{G}_3} \oplus \ldots$$

Boundary geometry of true flavor center symmetry Z_{G_F}

M. C., J. J. Heckman, M. Hübner and E. Torres: "0-Form, 1-Form and 2-Group Symmetries via Cutting and Gluing of Orbifolds," 2203.10102



[Mayer, 1929], [Vietoris, 1930]

- Key: Mayer-Vietoris sequence in homology for singular boundary $\partial X = \partial X^\circ \cup T_K$

true flavor center naïve flavor center

$Z_{\text{0-form}}$:
$$0 \to \ker(\iota_1) \to H_1(\partial X^\circ \cap T_K) \xrightarrow{\iota_1} \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to 0\,,$$

1-form:
$$0 \to \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to H_1(\partial X^\circ) \oplus H_1(T_K) \to H_1(\partial X) \to 0$$

naïve 1-form true 1-form

$$Z_{G_F} = \mathrm{Ker}\left(\iota_1 : Z_{\widetilde{G}_F} \cong H_1(\partial X^\circ \cap T_K) \to H_1(\partial X^\circ) \oplus H_1(T_K)\right)$$

Examples: noncompact orbifolds, elliptically fibered CY's etc. in 4,6 dim.

IV. Compact Models

- Compact singular space X → theory that includes quantum gravity & global symmetries gauged or broken
- What is M-theory gauge group?
- For elliptically fibered geometries via M/F-theory duality:

- Non-Abelian group algebras – ADE Kodaira classification

group topology → Mordell-Weil torsion

[Aspinwall, Morrison, 1998], [Mayrhofer, Morrison,Till, Weigand, 2014], [M.C., Lin, 2017]

- Abelian groups → Mordell-Weil ``free'' part

[Morrison, Park 2012], [M.C., Klevers, Piragua, 2013], [Borchmann, Mayrhofer, Palti, Weigand, 2013]…

- Total group topology → Shoida map of Mordell-Weil

[M.C., Lin, 2017]

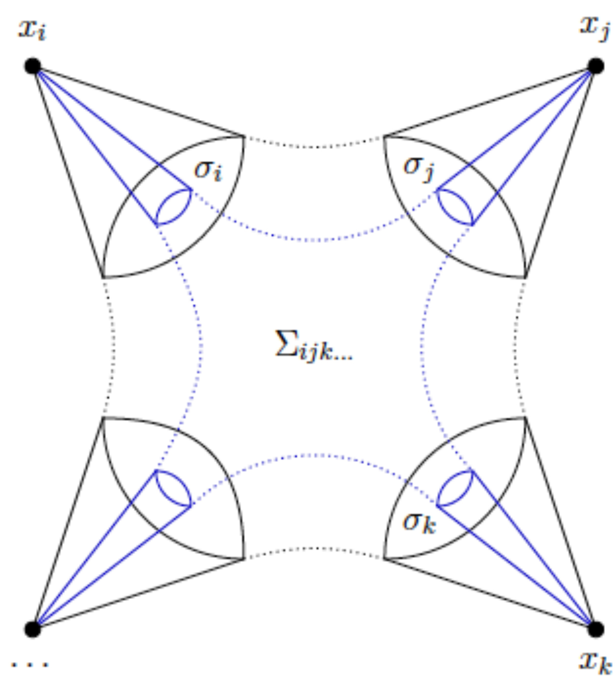
$$\frac{U(1)^r \times G_{\text{non-ab}}}{\prod_{i=1}^{r} \mathbb{Z}_{m_i} \times \prod_{j=1}^{t} \mathbb{Z}_{k_j}}$$

How to relate these results, encoded in resolution & arithmetic structure of elliptic curve, due Mordell-Weil, to higher-form symmetries and singular geometry?

Fate of higher-form structures via cutting and gluing

M. C., Heckman, Hübner, Torres,
Generalized Symmetries, Gravity, and the Swampland, 2307.1023

- Decompose $X \rightarrow \cup_i X_i^{loc}$ into local models X_i^{loc}
  Converse: Glue $\{X_i^{loc}\}$ to $X \Leftrightarrow$ Couple $\{SQFT_i\}$ to a resulting QFT & include gravity
  Schematically:



Defect Operators:
M2-/M5-brane wrapped on cones

Σ_{ijk} – two-chain glues defects

- Some relative Cycles in X_i^{loc} survive & compactify $\rightarrow$
  (Some) defects in $SQFT_i$ become dynamical - ``gauged''
  $\rightarrow$ determine the gauge group in compact models

Fate of higher-form structures in compact geometries (continued):

- Quantify: Mayer-Vietoris long exact sequence for covering $\{X_i^{loc}\}$ relates homologies of X_i^{loc} to X

$$\dots \xrightarrow{\jmath_n} H_n(X) \xrightarrow{\partial_n} H_{n-1}(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_{n-1}} H_{n-1}(X^\circ) \oplus H_{n-1}(X^{\mathrm{loc}}) \to \dots$$

w/ respect to the covering: $X = X^{\mathrm{loc}} \cup X^\circ\,, \qquad X^\circ = X \setminus X^{\mathrm{loc}}$

w/ total local model: $X^{\mathrm{loc}} = \coprod_i X_i^{\mathrm{loc}}$

- Example of K3 Surfaces → Mayer-Vietoris short exact sequence:

Extra $U(1)$'s | Massive Matter | Local Model Defect/Symmery Ops | Emergent/Broken Symmetries

$$0 \to H_2(X^\circ) \to H_2(X) \to H_1(\partial X^{\mathrm{loc}}) \to H_1(X^\circ) \to 0$$

$$H_1(\partial X^{\mathrm{loc}}) \cong \oplus_i H_1(\partial X_i^{\mathrm{loc}})$$

Decomposition of compact two-cycles into a sum of relative cycles associated with each local model ∂_2: $H_2(X) \to \bigoplus_i H_1(\partial X_i^{loc})$

Beyond K3 → subtleties

No time

V. Concluding Remarks

- Systematic determination of gauge group, including Abelian factors via cutting and gluing techniques
- Analysis of compact models, also beyond elliptically fibered K3, c.f., all T^4/Γ_i orbifolds:

$$T^4/\mathbb{Z}_2 : \qquad G = \frac{(SU(2)^{16}/\mathbb{Z}_2^5) \times U(1)^6}{\mathbb{Z}_2^6}$$

$$T^4/\mathbb{Z}_3 : \qquad G = \frac{(SU(3)^9/\mathbb{Z}_3^3) \times U(1)^4}{\mathbb{Z}_3^3}$$

$$T^4/\mathbb{Z}_4 : \qquad G = \frac{(SU(4)^4/\mathbb{Z}_4 \times \mathbb{Z}_2^2) \times SU(2)^6 \times U(1)^4}{\mathbb{Z}_4^2 \times \mathbb{Z}_2^2}$$

$$T^4/\mathbb{Z}_6 : \qquad G = \frac{([SU(6) \times SU(3)^4 \times SU(2)^5]/\mathbb{Z}_3 \times \mathbb{Z}_2) \times U(1)^4}{\mathbb{Z}_6^3 \times \mathbb{Z}_2}$$

& higher dimensional CY's: some T^6/Γ_i orbifolds (subtleties)..

- Open questions:
  Fate of 2-group → ``dissolved'' in global models
  Non-invertible symmetries → fusion algebra of TFT,
  starting w/local models…

Work in progress w/ J. Heckman, M. Hübner, E. Torres, H. Zhang

Thank you

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Congratulations to Chris on his 70th birthday
and to many more productive, scientific contributions!