W's, Branes and Trombones, but a missed dinner in Vonnas - physics adventures with Chris

K.S. Stelle

Imperial College London

Chrisfest

Firemeadow Lodge,
Cook's Branch, Montgomery County, Texas

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5 O'Clock and All is Well

In the spring of 1982, I was returning to Imperial College after a year-and-a-half European interlude first at CERN and then at the École Normale in Paris. I had been awarded an SERC 5-year fellowship and had to take it up within a narrow time window. I'm not sure exactly when Chris and I first met – perhaps it was in Cambridge during my earlier UK period from 1977 to 1980. But certainly we got to know each other at Imperial starting from the spring of 1982.

In those early years, of the people at Imperial, Chris worked with Mike Duff, Chris Isham and Bengt Nilsson but also with a number of colleagues all over the map (*e.g.* Gary Gibbons, Nick Warner, Don Page, . . .). One thing that I particularly remember, however, was the 5 O'Clock ritual, often with Bengt as Theory Group Town Crier. The Queens Arms pub, competed for equally by denizens of Imperial, of the Royal College of Art and of the Royal College of Music, became the venue for a more liquid context for physics discussions.

Queens Arms Center for Theoretical Physics

First Papers Together

It was in the context of the rebirth of string theory in 1986 that Chris and I really started working together, at times together with Mike Freeman, Martin Sohnius, Philip Candelas and Chris Hull, often on string counterterm structures:

C. N. Pope, M. F. Sohnius and K. S. Stelle, “Counterterm Counterexamples,” Nucl. Phys. B **283**, 192-204 (1987)

P. Candelas, M. D. Freeman, C. N. Pope, M. F. Sohnius and K. S. Stelle,
“Higher Order Corrections to Supersymmetry and Compactifications of the Heterotic String,”
Phys. Lett. B **177**, 341-346 (1986)

M. D. Freeman, C. N. Pope, M. F. Sohnius and K. S. Stelle,
“Higher Order σ Model Counterterms and the Effective Action for Superstrings,” Phys. Lett. B **178**, 199-204 (1986)

M. D. Freeman, C. N. Pope, C. M. Hull and K. S. Stelle,
“Space-time Versus World Sheet Supersymmetry in the Heterotic String,” Phys. Lett. B **185**, 351 (1987)

M. D. Freeman, C. N. Pope, M. F. Sohnius and K. S. Stelle, “Supersymmetry in Compactifications of the Heterotic String.”
Proc. 1986 Paris-Meudon Colloquium on String Theory, Quantum Cosmology and Quantum Gravity, 271-303

CERN, Trieste, the École Normale and elsewhere

In 1987 I was by then a Lecturer at Imperial. Chris had been awarded the same kind of SERC 5-year fellowship that I had and we all hoped that he would become a permanent member of the Imperial Theory Group as well. This was frustrated, dastardly, by USC, which grabbed him from us first. Before he left definitely for LaLa land, however, Chris and I both had a very productive time together at CERN in 1986-87.

One new direction was p-branes. The supergravity limit of superstring theory possesses it's field theory realisation of the eponymous super 1-brane, or superstring in 10 spacetime dimensions, as found by Dabholkar, Gibbons, Harvey and Ruiz-Ruiz. Once it was clear that such extended-object supergravity solutions were important elements of the solution-space, the search extended to other extended objects in other dimensions. What to do about these generalised p-branes at the quantum level became a vexing question, however. One approach was semiclassical: M. J. Duff, T. Inami, C. N. Pope, E. Sezgin and K. S. Stelle, "Semiclassical Quantization of the Supermembrane," Nucl. Phys. B **297**, 515-538 (1988)

That paper exploited a number of delightful concepts from the Hamiltonian formulation of general relativity and its evocation of Wheeler's "many-fingered time".

One feature which emerged from the semiclassical discussion, however, was an accumulation of an infinity of zero mode fluctuations in the membrane worldvolume analysis. How to think about this became clearer later on in the paper by Bernard de Wit, Martin Luscher and Hermann Nicolai which showed that the quantised membrane has an infinity of string-like excitations giving a continuous spectrum starting from zero. (Called by them an "instability", but what that means is that the supermembrane is really hairy with superstrings at the quantum level.)

This aspect of the membrane dynamics is not unrelated to the continuum of fluctuation modes in Kaluza-Klein systems with noncompact transverse spaces, where in general there is no mass gap.

W's and w's

After Chris moved from USC to Texas A&M in 1989, I had the pleasure to get to know a part of the US that I had been unfamiliar with, and also to get to know a dynamically evolving department. This development was also strongly supported by the generosity of the Mitchell Family Foundation. Among the new people I got to know were the impressive students whom Chris attracted, in particular Sean Shen and Hong Lü. Chris also continued to collaborate with Larry Romans back at USC. At about the same time, Ergin Sezgin came to A&M from the ICTP in Trieste, also continuing long-standing collaborations.

Topics of research interest in the early 1990's alternated between problems on the 2d worldvolume and problems in the effective supergravity theories (*i.e.* branes). On the 2d string worldvolume, W-algebra generalizations of the Virasoro algebra became a major topic, with the attendant symmetries for higher-spin excitations. One of our joint papers from this time was

E. Bergshoeff, P. S. Howe, C. N. Pope, E. Sezgin, X. Shen and K. S. Stelle, "Quantization deforms w(infinity) to W(infinity) gravity," Nucl. Phys. B **363**, 163-184 (1991)

Branes and their symmetries

In the mid 1990s, collaboration venues alternated between College Station, London, Trieste and Paris. Students, of course, graduate and move on. Hong Lü went to SISSA in Trieste, and later to the École Normale Supérieure in Paris. This led to a variety of new collaborations.

The supergravity brane spectrum became a major topic, and Chris and I collaborated on a good number of papers in this area. Together with Chris, Hong and Ergin we classified the families of brane solutions related by vertical and diagonal dimensional reduction, starting with a core model of gravity, a form field and a scalar

H. Lu, C. N. Pope, E. Sezgin and K. S. Stelle, "Stainless super p-branes," Nucl. Phys. B **456**, 669-698 (1995)

This work led to a set of related courses Chris and I gave at the 1996 ICTP summer school

H. Lu and C. N. Pope, "p-brane taxonomy," [arXiv:hep-th/9702086 [hep-th]].

K. S. Stelle, "Lectures on supergravity p-branes," [arXiv:hep-th/9701088 [hep-th]].

Hong, Eugène and the Trombone

When Hong Lü moved to the École Normale Supérieure in Paris, he and Chris and Eugène Cremmer wrote a series of very influential papers on the interrelations of supergravity dualities

E. Cremmer, B. Julia, H. Lu and C. N. Pope, "Dualization of dualities. 1.," Nucl. Phys. B **523**, 73-144 (1998)

E. Cremmer, B. Julia, H. Lu and C. N. Pope, "Dualization of dualities. 2. Twisted self-duality of doubled fields, and superdua

A related problem on which I worked there together with Chris, Hong and Eugène was to find the correct form of duality transformations generating orbits of form-field charged solutions (such as monopoles and branes) with a fixed asymptotic geometry and fixed asymptotic matter fields.

This was another aspect of the interrelation of supergravity and superstring duality symmetries

E. Cremmer, H. Lü, C.N. Pope and K.S.S., Nucl. Phys. B 520, 132 (1998)

D	G	H
9	$GL(2,\mathbb{R})$	$SO(2)$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$
7	$SL(5,\mathbb{R})$	$SO(5)$
6	$SO(5,5)$	$SO(5) \times SO(5)$
5	$E_{6(+6)}$	$USp(8)$
4	$E_{7(+7)}$	$SU(8)$
3	$E_{8(+8)}$	$SO(16)$

Supergravity symmetry groups.

These symmetries act not only on the supergravity form fields, however, but also on the G/H manifolds of scalar fields, changing also the asymptotic scalar values.

The key point to recognise is that for any of the maximally-noncompact supergravity symmetry groups G shown in the table, one has an Iwasawa decomposition of a general group element Λ, specialised to the vacuum point on the scalar modulus manifold G/H and to the charges Q defining a given fundamental soliton solution:

$$\Lambda = \mathbf{B}\,\mathbf{H} \ ,$$

where $\mathbf{H}$ is an element of the stability group H of the vacuum modulus point $\mathcal{M}_0$ on G/H and $\mathbf{B}$ is an element of the Borel subgroup corresponding to the charge Q, which has the property that it leaves Q invariant up to a rescaling.

So the basic idea is to first perform a standard supergravity Cremmer-Julia transformation that moves the charge Q to the intended target value on the solution orbit, but which also, unwantedly, moves the scalar field moduli (*i.e.* the scalar asymptotic values). The scalar moduli are then returned to their original values by the Borel transformation B. This also transforms the charges Q again, but now only by a rescaling.

The final step is to undo the rescaling of the charges Q by a symmetry transformation similar to the one in pure GR which moves the Schwarzschild solution along its orbit of different masses. One starts with the "trombone" symmetry of the equations of motion, under which the action is not invariant but scales homogeneously. In $D = 11$ supergravity, the transformations of the metric and 3-form field are

$$g_{\mu\nu} \longrightarrow \lambda^2 \, g_{\mu\nu} \ , \qquad A_3 \longrightarrow \lambda^3 \, A_3 \ ,$$

under which the Lagrangian scales as $\mathcal{L} \to \lambda^9 \, \mathcal{L}$.

Of course, such a trombone transformation also scales the asymptotic value of the metric, which is not wanted either. In order to return the asymptotic metric to its original value (*e.g.* Minkowski space at transverse infinity), one can however finish the construction with a simple general coordinate transformation.

This is how one generates *active* duality transformations moving a given starting brane or monopole solution along a charge orbit while holding all the asymptotic fields (the metric and the scalar moduli) constant. The trombone followed by the final general coordinate compensator leaves the asymptotic values of the metric and scalar fields unchanged, but it does change the mass or tension of the solution, so the active-symmetry orbit contains physically distinct solutions.

This is the classical situation. At the quantum level, Dirac quantisation restricts the $G(\mathbb{R})$ symmetry to $G(\mathbb{Z})$ and one should also be concerned about anomalies in the construction, especially possibly involving the trombone symmetry.

Other aspects of life

Overall, Chris and I have collaborated on 55 papers (but that's only 13% of the 421 papers listed for Chris by INSPIRE!). Some times we do other things, like hiking in the Maroon Bells area during an Aspen summer session

Or penetrate the lawless border zone at the Khyber Pass

Khyber Pass Desperados:
KSS, Eric Bergshoeff, Bryan Lynn, gunman, Carl Anderson, driver, Chris Pope

Or eat at good restaurants. And much of that has gone on also at Chris's house in College Station, where he himself is a fantastic cook. A number of times we made it to high level Michelen restaurants. But not always. There was one time when we had a reservation at Georges Blanc in Vonnas

But it was not to be. On the very day, we cancelled our reservation in order to finish a paper at CERN, showing our devotion to science above all. Precisely which paper it was is a bit hazy. And there are still some gastronomic recriminations. But it may have been

E. Bergshoeff, C. N. Pope and K. S. Stelle, "Covariant w(infinity) gravity," Phys. Lett. B **249**, 208-215 (1990).

In any case, I am honored to be here and to be able to give Chris my heartfelt congratulations on the midpoint of his scientific, culinary and other careers!

Congratulations Chris!