The Firewall Transformation: neither entirely consistent nor exactly canonical!

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Outline

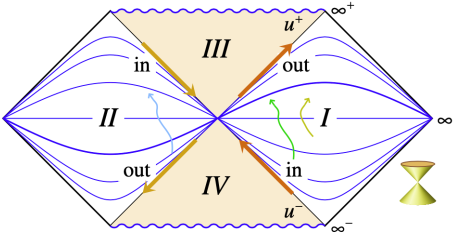
- What is a Firewall
- 't Hooft's proposed Firewall Transformation
- Casting it as a Canonical Transformation
- Accounting for Errors of Smallness
- Full Hamiltonian treatment
- Shifts nor really Canonical
- What can be done?

What is a Firewall

- Classical General Relativity suggests that nothing particularly remarkable happens in a sufficiently small neighborhood of a black hole horizon.
- Hawking radiation must lose energy as it escapes from close proximity to the black hole.
- Arbitrarily close to the black hole it must be arbitrarily energetic (even super-Planckian!).
- The original Firewall problem arises in close proximity to the black hole's future horizon.
- 't Hooft argues that Firewalls may exist at both the past horizon and the future horizon of an eternal black hole.

't Hooft's Proposal

In any Lorentz frame, 't Hooft distinguishes between soft and hard particles.

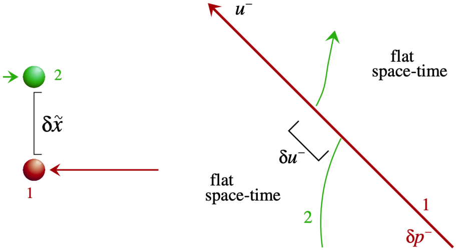


['t Hooft, Found Phys (2018) 48:1134-1149]

- The past Firewall represents the imploding matter which originally formed the black hole.
- The future Firewall represents very late and energetic Hawking particles (far from any vacuum state).
- Together, representing very large numbers of Quantum States, the Firewalls pose an unaddressed black hole information problem.
- Proceed by assuming their complete absence (ie, *"remove the firewall"*).

't Hooft's Proposal

We never encounter trans-Planckian particles in reality, so let's represent all pure quantum states of a black hole by allowing only soft particles in its environment.



['t Hooft, Found Phys (2018) 48:1134-1149]

- A spectator particle will appear to be dragged along after encountering a highly boosted particle:

$$\delta u^- = -\frac{4G}{c^3}\delta p^- \log|\delta\tilde{x}| \,.$$

- As gravity between soft particles is weak, standard quantum field theory and perturbative gravity apply.

- The footprints left by hard particles are themselves soft particles.

't Hooft's Firewall Transformation

- All ingoing particles encounter and interact with all outgoing particles.
- Consider a hard particle, momentum δp^-, from angular direction $\Omega = (\theta, \phi)$.
- It drags a soft particle, angular direction $\Omega' = (\theta', \phi')$, by an amount δu^-.
- Generalizing the above result, δu^- is given by:

$$\delta u^- = \frac{8\pi G}{c^3} f(\Omega', \Omega)\delta p^-, \quad \text{where} \quad (1 - \Delta_\Omega)\, f(\Omega', \Omega) = \delta^2(\Omega', \Omega),$$

  where Δ_Ω is the angular Laplacian.

- Summing over encounters, and "integrating", 't Hooft writes (and similarly for u^+ and p^+):

$$u^-(\Omega') = \frac{8\pi G}{c^3} \int d^2\Omega f(\Omega', \Omega) p^-(\Omega).$$

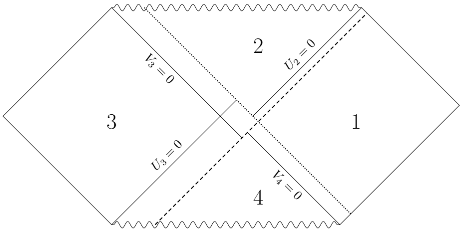
  where u^- and p^- are now also commuting quantum operators.

Casting 't Hooft's proposal in a Canonical Framework

't Hooft's result essentially eliminates half the degrees of freedom, since:

$$u^{\pm} \Leftrightarrow p^{\mp}, \quad \text{and} \quad [u^{-}, p^{+}] = [u^{+}, p^{-}] = i\hbar.$$

Note that 't Hooft's result is given in terms of quantum operators, but it has not been obtained from a Hamiltonian framework. We will use $u^{-} \Rightarrow \mathcal{U}$ and $u^{+} \Rightarrow \mathcal{V}$.



[[gr-qc] arXiv:2309.09891, 2309.09905]

- We work in the classical domain and develop a Hamiltonian perspective.
- We do simplify, replacing particles by null shells, with one intersection.
- We work first with the two shells, finding 't Hoofts setup inconsistent.
- Then with the full spacetime, we find his result is not canonical.

Developing the Spacetime Framework

- Each spacetime region is part of a Schwarzschild spacetime:

$$\mathrm{d}s^2 = -\Big(1 - \frac{2M_i}{R}\Big)\mathrm{d}T_i^2 + \Big(1 - \frac{2M_i}{R}\Big)^{-1}\mathrm{d}R^2 + R^2\mathrm{d}\Omega^2.$$

- Each metric can also be written in global, Kruskal coordinates:

$$\mathrm{d}s^2 = 2g_{\mathcal{U}_i\mathcal{V}_i}\mathrm{d}\mathcal{U}_i\mathrm{d}\mathcal{V}_i + R^2\mathrm{d}\Omega^2, \quad \text{where}$$

$$g_{\mathcal{U}_i\mathcal{V}_i} = 8M_i^2\frac{1 - 2M_i/R}{\mathcal{U}_i\mathcal{V}_i} = \frac{16M_i^3}{R}\mathrm{e}^{-R/2M_i}.$$

- In each region, these are related by:

$$\mathcal{U}_i\mathcal{V}_i = \Big(\frac{R}{2M_i} - 1\Big)\mathrm{e}^{R/2M_i}, \quad \text{and} \quad \mathcal{V}_i/\mathcal{U}_i = \mathsf{sign}\,\Big(\frac{R}{2M_i} - 1\Big)\mathrm{e}^{T_i/2M_i}.$$

- The energies of the shells are $E_{\text{in}} = M_1 - M_4$ and $E_{\text{out}} = M_4 - M_3$ as measured in region 4, and in region 2: $\tilde{E}_{\text{in}} = M_2 - M_3$ and $\tilde{E}_{\text{out}} = M_1 - M_2$.

Exact results, and Errors of Smallness

- We assume: $\mathcal{U}_{1,\text{out}} = \mathcal{U}_{2,\text{out}}$, $\mathcal{U}_{4,\text{out}} = \mathcal{U}_{3,\text{out}}$, $\mathcal{V}_{1,\text{in}} = \mathcal{V}_{4,\text{in}}$ $\mathcal{V}_{2,\text{in}} = \mathcal{V}_{3,\text{in}}$.
- Exact calculation gives (see also the Dray-'t Hooft-Redmount formula below):

$$\tilde{E}_{\text{in}} - E_{\text{in}} = \frac{2E_{\text{in}}E_{\text{out}}}{R_0 - 2M_4}, \quad \text{and} \quad E_{\text{out}} - \tilde{E}_{\text{out}} = \frac{2E_{\text{in}}E_{\text{out}}}{R_0 - 2M_4},$$

where R_0 is given by $\mathcal{U}_{i,\text{out}}\mathcal{V}_{i,\text{in}} = (R_0/2M_i - 1)\exp(R_0/2M_i)$.

- 't Hooft ignores the right hand sides, so he assumes $1 \gg \frac{2E_{\text{in/out}}}{R_0 - 2M_4} \sim \delta_{\text{in/out}}$.
- In his derivation, 't Hooft additionally assumes $\mathcal{U}_{i,\text{out}}$, $V_{i,\text{in}}$ are all small.
- Let $\mathcal{U}_{i,\text{out}} \sim \varepsilon_{\text{out}}$, $\mathcal{V}_{i,\text{in}} \sim \varepsilon_{\text{in}}$, then $|\mathcal{U}_{\text{out}}\mathcal{V}_{\text{in}}| = \left|\frac{R_0}{2M} - 1\right| \mathrm{e}^{R_0/2M} \sim \varepsilon_{\text{out}}\varepsilon_{\text{in}}$.
- Then, $E_{\text{in/out}} \sim \delta_{\text{in/out}}\varepsilon_{\text{in}}\varepsilon_{\text{out}}M$ and $\tilde{E}_{\text{in}} - E_{\text{in}} = \frac{2E_{\text{in}}E_{\text{out}}}{R_0 - 2M} \sim \delta_{\text{in}}\delta_{\text{out}}\varepsilon_{\text{in}}\varepsilon_{\text{out}}M$.
- Finally, note that $E_{\text{in}} \sim \delta_{\text{in}}\varepsilon_{\text{out}}M\mathcal{V}_{3,\text{in}}$ and $E_{\text{out}} \sim \delta_{\text{out}}\varepsilon_{\text{in}}M\mathcal{U}_{3,\text{out}}$.

Inexact results, and Errors of Smallness

- Direct calculation using the Kruskal coordinate conditions now gives:

$$\delta\mathcal{U}_{\text{out}} = \mathcal{U}_{1,\text{out}} - \mathcal{U}_{3,\text{out}} = -\frac{\mathrm{e}E_{\text{in}}}{M\mathcal{V}_{3,\text{in}}}\left(1 + O(\varepsilon_{\text{in}}\varepsilon_{\text{out}})\right) \sim \delta_{\text{in}}\varepsilon_{\text{out}},$$

$$\delta\mathcal{V}_{\text{in}} = \mathcal{V}_{1,\text{in}} - \mathcal{V}_{3,\text{in}} = -\frac{\mathrm{e}E_{\text{out}}}{M\mathcal{U}_{3,\text{out}}}\left(1 + O(\varepsilon_{\text{in}}\varepsilon_{\text{out}})\right) \sim \delta_{\text{out}}\varepsilon_{\text{in}},$$

  where we have dropped higher order terms in the shell energies.

- By also dropping the error terms shown, $O(\frac{r_0}{2M_i} - 1)$, we thus work to third order in $\varepsilon_{\text{in}}, \delta_{\text{in}}, \varepsilon_{\text{out}}, \delta_{\text{out}} \ll 1$.

- In his final step, 't Hooft assumes ingoing momenta start out at $p_{\text{in,init}} = 0$, and that the outgoing particles start out on the horizon: $\mathcal{U}_{\text{out,init}} \sim \varepsilon_{\text{out}} = 0$.

- Now taking $p_{\text{out,init}} = 0$ and $\mathcal{V}_{\text{in,init}} = 0$ would mean additionally that $\varepsilon_{\text{in}} = 0$.

- 't Hooft's treatment then appears inconsistent, as these conditions force both $\delta\mathcal{U}_{\text{out}} = 0$ and $\delta\mathcal{V}_{\text{in}} = 0$, and there is no remaining Firewall Transformation.

Developing the Hamiltonian Framework

- For spherical symmetry, an ADM approach admits this metric decomposition:

$$
\begin{aligned}
\mathrm{d}s^2 &= g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\
&= -N^2\mathrm{d}t^2 + \Lambda^2(\mathrm{d}r + N^r\mathrm{d}t)^2 + R^2\mathrm{d}\Omega^2 \\
&= -(N^2 - \Lambda^2 N^{r2})\mathrm{d}t^2 + 2\Lambda^2 N^r \mathrm{d}t\mathrm{d}r + \Lambda^2\mathrm{d}r^2 + R^2\mathrm{d}\Omega^2,
\end{aligned}
$$

where $N(t,r)$ and $N^r(t,r)$ are the lapse and shift, and $\Lambda(t,r)$ and $R(t,r)$ are the canonical variables of the metric. All are C^0 functions of r and t.

- Definition of the canonical momenta give:

$$
\dot{\Lambda} = N\left(\frac{\Lambda P_\Lambda}{R^2} - \frac{P_R}{R}\right) + (N^r\Lambda)', \quad \text{and} \quad \dot{R} = -\frac{NP_\Lambda}{R} + N^r R',
$$

- For a massless shell at $r = \mathfrak{r}(t)$, we find $\dot{\mathfrak{r}} = \eta\frac{N}{\Lambda} - N^r$ in terms of canonical variables, in which $\eta = \mathrm{sign}(\mathfrak{p})$ is the sign of the momentum $\mathfrak{p}(t)$ of the shell.
- The full action can be written in Hamiltonian form as:

$$
S = \int \mathrm{d}t\Big(\mathfrak{p}_{\text{in}}\dot{\mathfrak{r}}_{\text{in}} + \mathfrak{p}_{\text{out}}\dot{\mathfrak{r}}_{\text{out}} + \int dr(P_\Lambda\dot{\Lambda} + P_R\dot{R} - NH - N^r H_r)\Big),
$$

and Hamilton's equations of motion follow as usual.

Exploring the Hamiltonian Framework

Note:

$$H = \frac{\Lambda P_\Lambda^2}{2R^2} - \frac{P_\Lambda P_R}{R} + \frac{RR''}{\Lambda} - \frac{RR'\Lambda'}{\Lambda^2} + \frac{R'^2}{2\Lambda} - \frac{\Lambda}{2} + \frac{\eta_{\mathrm{in}}\mathfrak{p}_{\mathrm{in}}}{\Lambda}\delta(r-\mathfrak{r}_{\mathrm{in}}) + \frac{\eta_{\mathrm{out}}\mathfrak{p}_{\mathrm{out}}}{\Lambda}\delta(r-\mathfrak{r}_{\mathrm{out}}),$$

$$H_r = P_R R' - P_\Lambda'\Lambda - \mathfrak{p}_{\mathrm{in}}\delta(r-\mathfrak{r}_{\mathrm{in}}) - \mathfrak{p}_{\mathrm{out}}\delta(r-\mathfrak{r}_{\mathrm{out}}).$$

Then:

$$\dot{P}_\Lambda = \frac{1}{2}N\Big(-\frac{P_\Lambda^2}{R^2} - \Big(\frac{R'}{\Lambda}\Big)^2 + 1 + \frac{2\eta_{\mathrm{in}}\mathfrak{p}_{\mathrm{in}}}{\Lambda^2}\delta(r-\mathfrak{r}_{\mathrm{in}}) + \frac{2\eta_{\mathrm{out}}\mathfrak{p}_{\mathrm{out}}}{\Lambda^2}\delta(r-\mathfrak{r}_{\mathrm{out}})\Big) - \frac{N'RR'}{\Lambda^2} + N^r P_\Lambda',$$

$$\dot{P}_R = N\Big(\frac{\Lambda P_\Lambda^2}{R^3} - \frac{P_\Lambda P_R}{R^2} - \Big(\frac{R'}{\Lambda}\Big)'\Big) - \Big(\frac{N'R}{\Lambda}\Big)' + (N^r P_R)',$$

$$\dot{\mathfrak{p}} = -\mathfrak{p}\Big(\eta\frac{N}{\Lambda} - N^r\Big)'\Big|_{r=\mathfrak{r}},$$

which, along with $H=0$ and $H^r=0$, are the remaining Hamiltonian equations.

Exploring the Equations of Motion

- Off the shells, the equations of motion are the vacuum Einstein equations.
- The canonical variables Λ, R, and $\mathfrak{r}_{\text{in/out}}$ are all continuous across the shells.
- However, P_Λ and P_R are discontinuous across the shells.
- Then, $\dot{R}$ and $\dot{\Lambda}$ inherit discontinuities, as do R' and Λ'.
- Defining $\Delta_{\text{in/out}} f \equiv \lim_{\epsilon \to 0+} (f(t, \mathfrak{r}_{\text{in/out}} + \epsilon) - f(t, \mathfrak{r}_{\text{in/out}} - \epsilon))$, we find:

$$\Delta R' = -\frac{\eta \mathfrak{p}}{R}, \qquad \Delta_{\text{out}}(\Delta_{\text{in}} P_R) = 0,$$

$$\Delta P_\Lambda = -\frac{\mathfrak{p}}{\Lambda}, \qquad \Delta_{\text{out}}(\Delta_{\text{in}} \Lambda') = 0,$$

$$\Delta \Lambda' = \frac{\Lambda}{N} \Delta N' - \eta \frac{\Lambda^2}{N} \Delta N^{r\prime}, \qquad \Delta_{\text{out}}(\Delta_{\text{in}} P_\Lambda) = 0,$$

$$\Delta P_R = \eta \frac{R}{N} \Delta N' - \frac{\mathfrak{p}}{R}, \qquad \Delta_{\text{out}}(\Delta_{\text{in}} R') = 0, \quad \text{plus}$$

$$\Delta_{\text{out}}(\Delta_{\text{in}} N') = 0,$$

$$\Delta_{\text{out}}(\Delta_{\text{in}} N^{r\prime}) = 0,$$

and $\Delta_{\text{in}} \dot{\mathfrak{p}}_{\text{in}} = 0$, $\Delta_{\text{out}} \dot{\mathfrak{p}}_{\text{out}} = 0$, while
$\Delta_{\text{out}} \dot{\mathfrak{p}}_{\text{in}}(t_0) = 2\mathfrak{p}_{\text{in}} \Delta_{\text{out}} N^{r\prime}\Big|_{r=\mathfrak{r}_{\text{in/out}}(t_0)}$, $\Delta_{\text{in}} \dot{\mathfrak{p}}_{\text{out}}(t_0) = 2\mathfrak{p}_{\text{out}} \Delta_{\text{in}} N^{r\prime}\Big|_{r=\mathfrak{r}_{\text{in/out}}(t_0)}$.

Introducing Generalized Kruskal Coordinates

- The Kruskal coordinates of slide 8 do not match at the shell intersection.
- To correct, we use the rescaling freedom to define new Kruskal coordinates:

$$U_i = \sqrt{\frac{2M_i}{R_0}}\mathrm{e}^{(\tau_i - R_0)/4M_i}\mathcal{U}_i, \quad \text{and} \quad V_i = \sqrt{\frac{2M_i}{R_0}}\mathrm{e}^{-(\tau_i + R_0)/4M_i}\mathcal{V}_i$$

where the τ_i represent a residual shift freedom in the Schwarzschild times T_i.

- Then, at the collision, we find:

$$U_{\text{out}} V_{\text{in}} = 1 - \frac{2M}{R_0}, \quad \text{and} \quad \frac{V_{\text{in}}}{U_{\text{out}}} = \mathrm{e}^{(T_0 - \tau)/2M}$$

in each region separately, where $T_{0,i} = T_i(t_0, \mathfrak{r}(t_0))$, at the collision.

- Assuming $U_i = U_i(r,t)$ and $V_i = V_i(r,t)$, we can calculate directly:

$$\Delta\left(\frac{\dot{U}}{U'} + \frac{\dot{V}}{V'}\right) = 0, \quad \Delta\left(\frac{\dot{U}\dot{V}}{U'V'}\right) = 0, \quad \text{and} \quad \Delta\left(M^2 F \frac{U'}{U}\frac{V'}{V}\right) = 0,$$

and also find:

$$\dot{\mathfrak{r}}_{\text{in}} = -\frac{\dot{V}_{\text{i,in}}}{V'_{\text{i,in}}}, \quad \text{and} \quad \dot{\mathfrak{r}}_{\text{out}} = -\frac{\dot{U}_{\text{i,out}}}{U'_{\text{i,out}}},$$

which imply that $V_{\text{i,in}}$ and $U_{\text{i,out}}$ are constant along their respective shells.

Using Generalized Eddington-Finkelstein Coordinates

- From an Edington-Finkelstein-like embedding we additionally find:

$$\Delta_{\text{in}}\left(M\frac{V'}{V}\right) = 0 = \Delta_{\text{out}}\left(M\frac{U'}{U}\right).$$

- Then we can show:

$$\Delta_{\text{in}} R' = -\frac{4M}{R}\frac{\hat{V}'}{\hat{V}}\Delta_{\text{in}} M \quad \Longrightarrow \quad \mathfrak{p}_{\text{in}} = \eta_{\text{in}}\frac{4MV'_{\text{in}}}{V_{\text{in}}}\Delta_{\text{in}} M,$$

$$\Delta_{\text{out}} R' = -\frac{4M}{R}\frac{\hat{U}'}{\hat{U}}\Delta_{\text{out}} M \quad \Longrightarrow \quad \mathfrak{p}_{\text{out}} = \eta_{\text{out}}\frac{4MU'_{\text{out}}}{U_{\text{out}}}\Delta_{\text{out}} M.$$

- Shell momenta are now directly related to their energies.
- We can combine with earlier results to now obtain:

$$\Delta_{\text{out}}\left(MF\frac{V'}{V}\right) = 0 = \Delta_{\text{in}}\left(MF\frac{U'}{U}\right).$$

- Consistency then implies the Dray-'t Hooft-Redmount result:

$$\left(1 - \frac{2M_1}{R_0}\right)\left(1 - \frac{2M_3}{R_0}\right) = \left(1 - \frac{2M_2}{R_0}\right)\left(1 - \frac{2M_4}{R_0}\right).$$

The ADM analogue of the Firewall Transformation

- The Dray-'t Hooft-Redmount result also implies:

$$\frac{E_{\text{in}}}{1-2M_4/R_0}=\frac{\tilde{E}_{\text{in}}}{1-2M_3/R_0},\quad \frac{E_{\text{in}}}{1-2M_1/R_0}=\frac{\tilde{E}_{\text{in}}}{1-2M_2/R_0},$$

$$\frac{E_{\text{out}}}{1-2M_4/R_0}=\frac{\tilde{E}_{\text{out}}}{1-2M_1/R_0},\quad \frac{E_{\text{out}}}{1-2M_3/R_0}=\frac{\tilde{E}_{\text{out}}}{1-2M_2/R_0}.$$

- We can also show:

$$V_{1,\text{in}}-V_{2,\text{in}}=-\frac{1}{2M_1R_0}\frac{\eta_{\text{out}}\mathfrak{p}_{\text{out}}}{U'_{1,\text{out}}},\qquad U_{1,\text{out}}-U_{4,\text{out}}=-\frac{1}{2M_1R_0}\frac{\eta_{\text{in}}\mathfrak{p}_{\text{in}}}{V'_{1,\text{in}}},$$

$$=\qquad\qquad\qquad\qquad =$$

$$V_{4,\text{in}}-V_{3,\text{in}}=-\frac{1}{2M_4R_0}\frac{\eta_{\text{out}}\mathfrak{p}_{\text{out}}}{U'_{4,\text{out}}},\qquad U_{2,\text{out}}-U_{3,\text{out}}=-\frac{1}{2M_2R_0}\frac{\eta_{\text{in}}\mathfrak{p}_{\text{in}}}{V'_{2,\text{in}}}.$$

- These ADM analogues of 't Hooft's Firewall Transformation are exact.
- The LHS are in terms of Kruskal, not embedded shell, coordinates.
- They are not canonical, so they are not really suitable to be quantized.
- Note: $U_{i,\text{out}}$, $V_{i,\text{in}}$, $\mathfrak{p}_{\text{out}}/U'_{i,\text{out}}$, and $\mathfrak{p}_{\text{in}}/V'_{i,\text{in}}$ are all constants of the motion.

What we have done so far

Our primary results are

- The various $\Delta_{\mathrm{out}}(\Delta_{\mathrm{in}}(X)) = 0$ equations.
- The $\Delta(\dot{\mathfrak{p}})$ equations for the shell's momenta.
- The several consequences of the Dray-'t Hooft-Redmount formula, as used in constructing the classical analogue of the Firewall Transformations.
- The exact shift equations on the previous slide, which are completely coordinate independent, and completely free of approximation.
- We have provided a general framework suitable for investigating canonical quantization.
- We have kept the radial coordinate r of the foliation completely arbitrary.
- Seen that quantization of shells/particles described by different spacetime coordinates will result in different quantum theories.

Restricted Shell Hamiltonian

We use hybrid coordinates, with the Schwarzschild time T as the time coordinate t, and the Kruskal coordinates $\mathcal{U}$ or $\mathcal{U}$ as the foliation radial coordinate r.

- The metrics are:

$$\begin{aligned} \mathrm{d}s^2 &= 2e^{-T/2M} g_{\mathcal{U}\mathcal{V}} (\mathrm{d}\mathcal{V}^2 - \tfrac{\mathcal{V}}{2M} \mathrm{d}T \mathrm{d}\mathcal{V}) + R^2 \mathrm{d}\Omega^2 \\ &= 2e^{T/2M} g_{\mathcal{U}\mathcal{V}} (\mathrm{d}\mathcal{U}^2 + \tfrac{\mathcal{U}}{2M} \mathrm{d}T \mathrm{d}\mathcal{U}) + R^2 \mathrm{d}\Omega^2. \end{aligned}$$

Then:

$$\begin{aligned} \mathcal{H}_{\mathcal{V}} &= \mathfrak{p} \left(\eta \frac{\hat{N}}{\hat{\Lambda}} - \hat{N}^r \right) \\ &= \frac{1}{2M} \frac{\eta\varepsilon + 1}{2} \mathfrak{p} \hat{\mathcal{V}}, \\ \mathcal{H}_{\mathcal{U}} &= \mathfrak{p} \left(\eta \frac{\hat{N}}{\hat{\Lambda}} - \hat{N}^r \right) \\ &= \frac{1}{2M} \frac{\eta\varepsilon - 1}{2} \mathfrak{p} \hat{\mathcal{U}}. \end{aligned}$$

So:

$$\begin{aligned} \mathcal{H}_{\mathcal{U}\mathrm{in}} &= -\frac{1}{2M} \mathfrak{p}_{\mathcal{U}\mathrm{in}} \mathfrak{r}_{\mathcal{U}_{\mathbf{in}}}, \\ \mathcal{H}_{\mathcal{V}\mathrm{in}} &= 0, \\ \mathcal{H}_{\mathcal{U}\mathrm{out}} &= 0, \\ \mathcal{H}_{\mathcal{V}\mathrm{out}} &= \frac{1}{2M} \mathfrak{p}_{\mathcal{V}\mathrm{out}} \mathfrak{r}_{\mathcal{V}_{\mathbf{out}}}. \end{aligned}$$

where $\hat{X} = X(r = \mathfrak{r})$, and ε is the sign of $\hat{\mathcal{U}}$ or $\hat{\mathcal{V}}$.

The Firewall Transformation as a canonical transformation

- Note that:

$$\mathcal{H}_c(\mathcal{U}_{\rm in}, p_{\mathcal{U}_{\rm in}}, \mathcal{V}_{\rm out}, p_{\mathcal{V}_{\rm out}}) = \frac{p_{\mathcal{V}\rm out}\mathcal{V}_{\rm out}}{2M} - \frac{p_{\mathcal{U}\rm in}\mathcal{U}_{\rm in}}{2M}, \quad \text{while}$$
$$\mathcal{H}_c(\mathcal{U}_{\rm out}, p_{\mathcal{U}_{\rm out}}, \mathcal{V}_{\rm in}, p_{\mathcal{V}_{\rm in}}) = 0,$$

  since, in the latter case, all canonical shell variables are constant.

- Confining ourselves to 't Hooft's (near horizon) firewall formulation, we find:

$$\begin{aligned} \mathcal{U}_{2,\rm out} &= \mathcal{U}_{4,\rm out} + \frac{e}{4M_4^2} p_{4,\mathcal{V}\rm in}, \\ \mathcal{V}_{2,\rm in} &= \mathcal{V}_{4,\rm in} + \frac{e}{4M_4^2} p_{4,\mathcal{U}\rm out}, \\ p_{\mathcal{U}_{2,\rm out}} &= p_{\mathcal{U}_{4,\rm out}}, \\ p_{\mathcal{V}_{2,\rm in}} &= p_{\mathcal{V}_{4,\rm in}}, \end{aligned} \qquad \text{and} \qquad \begin{aligned} \mathcal{U}_{1,\rm out} &= \mathcal{U}_{3,\rm out} + \frac{e}{4M_3^2} p_{1,\mathcal{V}\rm in}, \\ \mathcal{V}_{3,\rm in} &= \mathcal{V}_{1,\rm in} + \frac{e}{4M_3^2} p_{3,\mathcal{U}\rm out}, \\ p_{\mathcal{U}_{1,\rm out}} &= p_{\mathcal{U}_{3,\rm out}}, \\ p_{\mathcal{V}_{3,\rm in}} &= p_{\mathcal{V}_{1,\rm in}}, \end{aligned}$$

  are both canonical transformations, in which the new variables are now continuous with their counterparts at the shell intersection.

- These transformations each serve different roles in 't Hooft's framework.

What can be done

- Careful analysis of 't Hooft's work will help throw more light on what he proposes.
- Keeping shells off their respective horizons prevents the Firewall Transformation from becoming degenerate.
- A canonical transformation provides a clear way of interpreting the Firewall Transformation.
- The distinction between hard and soft particles warrants further analysis.
- Quantization of our results may confirm the Firewall Transformation, or it may offer a meaningful alternative.
- An alternative analysis could determine if the quantum Firewall Transformation will resolve the black hole information paradox.